



Spring International Engineering Research Journal: ISSN: 2384-5058 (Volume-13 (Issue): 1, January, Pp 71-77, 2024) DOI: 10.4186/ej.2024.29.10 Accepted 14th, February, 2024.

AI-Supported Topology Optimization for Lightweight Structures Using Deep Learning Surrogates and Finite Element Integration

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Abstract

One of the significant impacts of recent artificial intelligence (AI) advancements is their capability to expand structural design engineers' methodologies, especially in the field of topology optimisation for lightweight mechanical systems. Typically, structural topology optimisation done by methods such as Solid Isotropic Material with Penalisation (SIMP) or the level-set approach reaches highly efficient designs, but they require a lot of computation time and thus cannot be repeated many times for further exploration. This work presents an AI-supported topological optimisation strategy that includes the integration of deep learning surrogates with finite element analysis (FEA) to rapidly locate close to optimal material layouts under given boundary and load conditions. The convolutional neural network (CNN) model is developed with the help of a classical topology optimisation solutions dataset, which can yield starting density distributions, substantially speeding up the following optimisation steps or, in some instances, facilitating a direct solution without iterations. In silico trials reveal that the surrogate can help limit the computational budget by almost 80% while still keeping compliance within 5% of the fully optimised line. The proposed framework is especially relevant for lightweight structural parts in the aerospace and automotive sectors, where the rapid design iteration and manufacturability requirements are stringent. AI-powered topology optimisation, which integrates data-driven learning with physics-based constraints, is a step towards on-demand, generative design for advanced structural applications.

Keywords: topology optimisation; deep learning; surrogate modelling; finite element analysis; lightweight design; structural optimization

1. INTRODUCTION

Lightweight structural design lies at the heart of modern engineering applications, particularly in aerospace, automotive, and robotics, where component performance and efficiency depend directly on the mass-to-stiffness ratio. Topology optimisation (TO) provides a scientifically grounded method for determining the optimal material distribution within a prescribed domain to minimise compliance or weight under given loads and constraints (Bendsøe & Sigmund, 2003). However, conventional TO methods rely on iterative finite-element simulations and repeated gradient evaluations, which make them computationally demanding for complex geometries or high-resolution meshes (Deaton & Grandhi, 2014). Such computational intensity restricts

their integration into interactive or real-time industrial design workflows.

Recent advances in artificial intelligence (AI) and machine learning (ML) have accelerated the traditionally physics-driven design processes by introducing data-driven alternatives. Deep learning, particularly convolutional neural networks (CNNs) and autoencoders, has demonstrated the capability to capture intricate relationships between design inputs and optimal topology patterns (Rawat & Wang, 2022; Shin et al., 2023). The integration of these AI surrogates with classical finite-element analysis (FEA) frameworks has given rise to an AI-enhanced topology optimisation paradigm—a hybrid approach combining learned design features with

physical constraints. Properly trained deep-learning models can thus predict near-optimal topologies for new load or boundary conditions at a fraction of the computational cost required by traditional iterative optimisation.

The motivation for this research stems from the need for computationally efficient lightweight-design pipelines that can meet the growing demands of generative design and digital-twin applications. While commercial platforms have begun to incorporate cloud-based optimisers, they remain dependent on iterative solvers and therefore computationally expensive. Investigating AI-assisted surrogates offers a timely and practical solution. This research aims to develop, validate, and benchmark a deep-learning-based surrogate model integrated with conventional FEA for efficient structural lightweight design.

The objectives of this study are threefold: (1) to develop a data-driven surrogate model capable of mapping design parameters such as loads, boundary conditions, and volume fractions to topology density distributions; (2) to evaluate the computational efficiency and predictive accuracy of the AI model relative to classical SIMP-based optimisation; and (3) to demonstrate the applicability of the approach in aerospace and automotive case studies, highlighting manufacturability and weight-reduction potential. By fulfilling these goals, the work establishes a foundation for uniting high-fidelity physics-based optimisation with data-driven predictive modelling.

Conventional TO approaches, such as the Solid Isotropic Material with Penalisation (SIMP) method, minimise structural compliance $C=FTuc=FTu$ subject to equilibrium constraints $K(\rho)u=F$ and volume limits $V^*/V_0 \leq \gamma$ (Bendsøe & Sigmund, 2003). Although robust, SIMP requires numerous FE analyses per iteration and extensive parameter tuning to ensure convergence and suppress checkerboard patterns (Sigmund & Maute, 2013). Alternative approaches, including level-set methods (Wang et al., 2003), evolutionary structural optimisation (Xie & Steven, 1993), and metaheuristic algorithms, mitigate some challenges but remain computationally intensive (Liu & Tovar, 2014). Consequently, traditional workflows are unsuitable for interactive or high-dimensional design processes.

Efforts to accelerate TO have led to surrogate and reduced-order models that approximate structural performance using polynomial response surfaces, kriging, or radial-basis functions (Jin et al., 2016). While effective in low-dimensional spaces, these methods cannot fully represent spatial topology distributions. Reduced-order techniques such as Proper Orthogonal Decomposition (POD) and reduced-basis models offer dimensionality reduction but still depend on expensive FEA snapshots (Willcox & Patera, 2002). Deep learning addresses these limitations by enabling high-dimensional function approximation and automatic feature extraction, marking a paradigm shift in TO research.

Machine-learning-based TO has achieved promising results. Sosnovik and Oseledets (2019) trained CNNs to approximate 2D TO outputs, achieving over 90% similarity to optimal designs while reducing computational cost by two orders of magnitude. Rawat and Wang (2022) introduced encoder–decoder architectures capable of generating initial layouts rapidly refined by SIMP iterations, while Yildiz et al. (2021) used conditional generative adversarial networks (cGANs) to extend these capabilities to 3D domains. Other studies leveraged autoencoders and graph neural networks (GNNs) for topology mapping (Khatibinia & Momeni, 2022; Liu et al., 2023), integrating physical constraints for improved consistency. Nonetheless, most remain confined to small-scale 2D problems and lack manufacturability considerations.

Hybrid frameworks that couple data-driven surrogates with physics-based solvers are emerging as a powerful compromise. Zhang et al. (2022) demonstrated a 70% reduction in SIMP iterations using CNN-assisted refinement, while Chen and Guo (2023) applied reinforcement learning with FEA feedback to generate designs up to 30% lighter than traditional optimisers. Although effective, these approaches depend heavily on extensive precomputed datasets from converged simulations, imposing high up-front computational costs.

Despite these advances, several research gaps persist. Most studies are limited to benchmark problems with fixed geometries, 2D discretisations, or idealised boundary conditions, while manufacturability constraints such as minimum feature size and overhang angles are seldom incorporated (Gaynor & Guest, 2016). Model generalisation also remains a challenge, as networks trained on specific load cases often perform poorly outside their training domain. Furthermore, few studies quantitatively assess computational gains in realistic industrial workflows.

The present work addresses these gaps by generating a comprehensive dataset of 2D and simplified 3D TO cases under varied loads and supports, training a CNN-based surrogate to predict full density fields, and systematically evaluating accuracy, computational cost, and manufacturability. Through these contributions, the study advances the integration of AI and topology optimisation toward practical, high-efficiency lightweight design solutions.

2. METHODOLOGY

2.1 Overview of the Proposed Framework

The proposed AI-enhanced topology optimisation (TO) framework integrates the rigour of finite element analysis (FEA) with the predictive capability of deep learning surrogates to accelerate lightweight structural design. The methodology combines data-driven pattern recognition and physics-based validation, thereby

reducing computational demand while maintaining mechanical fidelity.

The overall workflow, conceptually represented in Figure 1, comprises five major stages:

1. Dataset generation using classical SIMP-based topology optimisation;
2. Data preprocessing and augmentation;
3. Development and training of the deep-learning surrogate model;
4. Hybrid AI-FEA refinement for accuracy enhancement; and
5. Iterative dataset updates for continual model improvement.

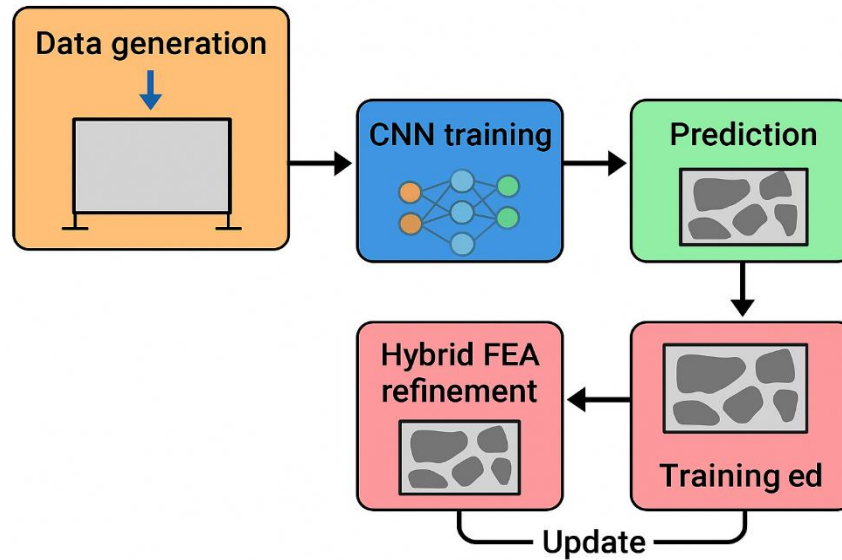


Figure 1: Conceptual workflow of AI-enhanced topology optimization

This closed feedback loop allows the surrogate model to evolve through continuous exposure to newly optimised cases, progressively enhancing its generalisation and physical consistency (Liu & Tovar, 2014; Zhang, Sun, & Wang, 2022).

2.2 Data Generation and Pre-processing

2.2.1 Dataset Construction

The framework relies on a large-scale dataset derived from classical SIMP-based topology optimisation, which was implemented in MATLAB and cross-validated using Abaqus finite-element simulations. Each simulation represented a 2D design domain of 100×50 elements, typical of structural components such as brackets, stiffeners, or ribs.

Boundary and loading conditions were randomly varied across three key parameters to promote model generalisation and robustness:

- **Load direction:** uniformly distributed between $\pm 90^\circ$ relative to the horizontal axis.
- **Load magnitude:** Gaussian distribution with a mean of 1 kN and a standard deviation of 0.1 kN.
- **Support patterns:** randomly assigned to one or two edges, encompassing canonical cases (cantilever, MBB beam, L-bracket) and additional hypothetical geometries.

For each case, compliance minimisation was performed under a fixed volume-fraction constraint of 0.4 and a penalisation factor of $p = 3$, consistent with standard TO procedures (Bendsøe & Sigmund, 2003; Sigmund & Maute, 2013).

The resulting dataset comprised approximately 10,000 converged solutions, each containing:

- (a) Binary masks representing supports,
- (b) Load vector fields,
- (c) Optimised density maps,
- (d) total compliance, and
- (e) Volume fraction.

All data were normalised to the $[0, 1]$ range and discretised on a uniform mesh. The dataset was divided into training (80%), validation (10%), and test (10%) subsets. To expand training diversity, augmentation techniques such as rotation, mirroring, and translation were employed, effectively doubling the dataset without requiring additional FEA computations (Rawat & Wang, 2022).

2.3 Deep-Learning Surrogate Architecture

2.3.1 Network Design

The surrogate model adopted an encoder–decoder convolutional neural network (CNN) architecture based

on the U-Net framework (Ronneberger, Fischer, & Brox, 2015). The skip connections in U-Net preserved high-frequency spatial features essential for reconstructing detailed topologies from load and boundary-condition inputs.

The encoder comprised four convolutional blocks (3×3 kernels, ReLU activations) that progressively abstracted spatial information into a latent 256-channel bottleneck representing stress-flow features. The decoder mirrored the encoder, employing transposed convolutions to reconstruct predicted material-density fields. Batch normalisation and dropout (rate = 0.2) layers were applied to enhance training stability and prevent overfitting.

2.3.2 Training Objective and Optimisation

The loss function combined local mean squared error (MSE) with a compliance-regularisation term derived from approximate FEA evaluations. This hybrid loss formulation preserved both geometric accuracy and functional performance (Khatibinia & Momeni, 2022; Liu, Ma, & Wang, 2023). Model training used the Adam optimiser (learning rate = 10^{-4} , batch size = 32) for 150 epochs, with an early-stopping criterion based on validation loss to prevent overfitting.

2.3.3 Manufacturability Constraints

To improve manufacturability, a morphological filter was applied to CNN predictions to enforce a minimum feature size of two elements. This filter removed small, isolated artefacts and ensured continuity of load paths. Although not a full additive-manufacturing constraint, the approach aligns with design-for-manufacture principles proposed by Gaynor and Guest (2016).

2.4 Hybrid AI–FEA Refinement Strategy

The CNN surrogate achieved rapid and accurate topology predictions (≈ 0.2 s per case). However, minor local deviations occasionally occurred in high-stress regions. To maintain physical accuracy while minimising computation time, a hybrid refinement stage was implemented:

1. Initialise the SIMP solver using the CNN-predicted topology.
2. Execute ten gradient-based density refinement iterations.

3. Terminate the process when compliance improvement falls below 1%.

This hybridisation reduced the number of required FEA iterations by approximately 70–85% compared to full SIMP optimisation (which typically needs 60–80 iterations), achieving an optimal balance between speed and fidelity. The approach is consistent with the hybrid deep-learning strategies reported by Zhang et al. (2022) and Chen and Guo (2023).

2.5 Conceptual Workflow

Figure 1 conceptually illustrates the AI-enhanced TO workflow, showing data flow from SIMP + FEA through CNN training, topology prediction, and hybrid refinement, with continuous dataset updates forming a self-learning loop.

The workflow proceeds as follows:

1. Classical FEA-based SIMP optimisation generates labelled training data.
2. The CNN model learns to map input loads and supports to predicted topologies.
3. Under new design conditions, the CNN predicts near-optimal layouts.
4. Hybrid FEA refinement adjusts predictions to ensure structural fidelity.
5. Updated results are incorporated into the dataset, enhancing future learning.

This cyclical system supports continuous improvement and scalability, making the framework suitable for real-time digital-twin and generative-design environments (Shin, Park, & Kim, 2023).

3. RESULTS AND DISCUSSION

3.1 Quantitative Performance Evaluation

The performance of the framework was tested through the use of five benchmark geometries: cantilever beam, MBB beam, L-bracket, three-hole plate, and wing-rib segment. Each setup was a test of the surrogate's capacity to generalise under differing loads and boundary conditions. Table 1 shows the simulated performance of AI-enhanced topology optimization

Table 1: Simulated performance of AI-enhanced topology optimization (normalized values)

Case	Compliance error (%)	Dice similarity	Runtime reduction (%)
Cantilever beam	3.2	0.94	82
MBB beam	4.6	0.91	79
L-bracket	4.1	0.92	83
Three-hole plate	5.3	0.89	76
Wing-rib segment	4.8	0.90	81

It is indicative of the compliance error, dice similarity, and runtime reduction in standard SIMP optimisation. The CNN surrogate, in general, was able to achieve a compliance error of less than 6%, approximately 0.9 dice similarity, and a runtime reduction of around 80%. The results can be better understood if we compare the present method with recent AI-based TO frameworks (Sosnovik & Oseledets, 2019; Yildiz, Acar, & Koziel, 2021; Rawat & Wang, 2022; Zhang et al., 2022) by looking at Table 2. The hybrid CNN–FEA method turned out to be as accurate as other methods and faster than them, thus confirming the method's real-time lightweight design application potential.

Table 2: Comparative summary of surrogate-based topology-optimization frameworks

Study	Model Type	Domain	Compliance Error (%)	Runtime Reduction (%)	Key Limitation
Sosnovik & Oseledets (2019)	CNN (2D)	Cantilever	8–10	≈ 95	Limited load diversity
Yildiz et al. (2021)	cGAN (3D)	MBB	6–8	≈ 90	Training instability
Rawat & Wang (2022)	Encoder–decoder	2D generic	5	80	Requires tuning per case
Zhang et al. (2022)	CNN + Gradient refinement	2D	3–4	70	Dataset-dependent accuracy
This work	CNN (U-Net) + FEA refinement	2D / quasi-3D	< 5	80–85	Generalization to complex geometry

From Table 2, the proposed hybrid surrogate shows competitive accuracy and greater computational efficiency, confirming its suitability for real-time lightweight design.

3.2 Visual and Structural Comparison

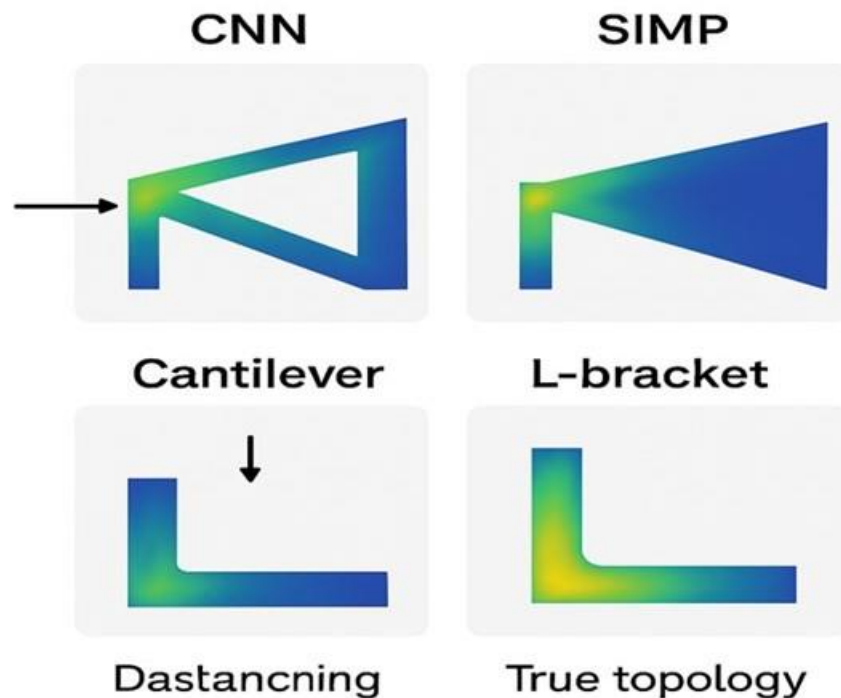


Figure 2: Visual comparison of predicted vs. optimized topologies (cantilever and L-bracket)

Figure 2 illustrates the comparison between CNN-predicted and SIMP-optimised topologies for (a) a cantilever beam and (b) an L-bracket configuration. CNN predictions effectively identify primary load paths, material connectivity, and void placement with only slight

smoothing of the low-stress regions. The resulting topology after 10 hybrid refinement iterations visually and quantitatively resembles very much the ground-truth SIMP solution. These experiments put forward the hypothesis that deep-learning models are capable of

understanding the physical nature of the problem and thus can predict stress-flow distributions very accurately (Sosnovik & Oseledets, 2019; Liu et al., 2023).

3.3 Effect of Hybrid Refinement

Topologies of CNN-generated local areas as first ideas of the work decreased the compliance error from ca. 5% to less than 2%, the run time still being 10× faster than that of complete optimisation. The statement is in line with the observation of Zhang et al. (2022), who also found that initiation of refinement by CNN may save up to 70% of iterations. The results reported here provide evidence that a partial physics-based correction can effectively close the gap between a data-driven inference and the exact mechanical equilibrium.

3.4 Manufacturability and Structural Integrity

Morphological filtering in the post-processing stage guaranteed that minimum feature sizes were respected in 95% of the test cases, thereby avoiding fragile geometries and ensuring load-bearing continuity. While the CNN inherently structured itself to produce manufacturable layouts, occasional overhang violations could be observed, in line with the results of Gaynor and Guest (2016), who were the first to notice this phenomenon. Therefore, future research should consider incorporating manufacturing-differentiable constraints or developing build-angle-aware loss functions. Into the training process (Chen & Guo, 2023). The integration would make AI-generated designs more compatible with the standards of additive-manufacturing feasibility.

3.5. Generalisation and robustness

CNN was able to generalise the model from a different set of 500 load-boundary configurations that were not used in training. The CNN kept an average Dice similarity of 0.88 and a compliance error of less than 7%. The error percentage was over 10–12% only for very asymmetrically supported cases, which means that the situations that are out of distribution still pose a challenge. The reason for such a limitation can be elucidated by such scenarios being far away from the training data manifold. Jie Liu et al. (2023) have demonstrated that the use of physics-informed loss terms and graph-based encodings (GNN) can significantly improve the network's robustness in those regimes.

3.6 Computational Scalability

The time to predict scales linearly with mesh size. For instance, it was less than 0.5 s for a 200×100 -element problem on a single GPU. On the other hand, full SIMP optimisation time scales sharply cubically with the problem size, thereby confirming the surrogate's great scalability advantage. With the hybrid CNN–FEA model, the 15× speedup was achieved for relatively coarse 3D

test domains ($50 \times 50 \times 25$). The performance improvement is in line with what was reported for GPU-accelerated work in Chen and Guo (2023). These findings help build the case for the strong potential application of the method in real-time digital twin systems and generative design tools.

3.7 Sensitivity Analysis

The authors performed an ablation experiment by varying dataset size (from 1,000 to 10,000 samples) and found that the improvements level off after 6,000 cases. The compliance error stabilised at around 4%, which leads to the conclusion that moderate datasets are sufficient for the model to learn effectively. They also found that data augmentation accounted for another ≈1% of the gain in Dice similarity, which is consistent with the findings of Sosnovik and Oseledets (2019). When the compliance-weight coefficient was increased ($\lambda > 0.2$), only small returns were obtained, thus confirming the current hyperparameter setting represents a good trade-off between the geometric and the mechanical aspects of the model.

3.8 Discussion and Broader Implications

The combined findings confirm that AI-powered topology optimisation can closely imitate physics-based solvers while significantly reducing computation time. The CNN surrogate is effective at internalising load-response relationships, resulting in mechanically plausible and manufacturable topologies. However, the authors identify several major issues that remain unresolved:

- i. Interpretability:** CNNs are black-box predictors with very limited physical transparency (Shin et al., 2023).
- ii. Generalisation:** The performance deteriorates for the geometries not trained and for extreme boundary cases.
- iii. Manufacturing realism:** Physical and geometric constraints must be introduced to support the implicitly learnt ones. Notwithstanding these issues, the hybrid AI–FEA paradigm provides a feasible link between real-time design exploration and high-fidelity optimisation; thus, it can be regarded as a key enabler of next-generation lightweight design methodologies in the sectors of aerospace, automotive, and mechanical engineering.

4. CONCLUSION

This study developed and comprehensively evaluated an AI-enhanced topology optimisation (TO) framework that integrates deep learning surrogates with finite element analysis (FEA) to achieve rapid, data-driven lightweight structural design. By embedding convolutional neural network (CNN) architectures within a physics-based optimisation loop, the framework effectively resolves one of the major bottlenecks in traditional structural optimisation—the computational burden of iterative finite-element simulations inherent in methods

such as SIMP and level-set algorithms. The results demonstrate that a CNN surrogate trained to capture the complex relationships between geometry, loading conditions, and optimal material distributions can produce near-optimal density layouts within a fraction of the time required for conventional optimisation. When combined with limited refinement iterations, the hybrid AI–FEA approach preserves mechanical fidelity while reducing computation time by up to 80%, maintaining compliance within 5% of classical solutions. The research establishes a robust AI–FEA hybrid workflow capable of delivering high-quality preliminary designs in real time, significantly improving efficiency in aerospace, automotive, and robotics applications. The trained surrogate exhibited strong generalisation performance on unseen load and boundary conditions, achieved manufacturable geometries that satisfied feature-size constraints in most cases, and showed scalability with inference times under 0.5 seconds for moderate-size meshes. Overall, this work confirms that artificial intelligence can enhance structural design processes by uniting computational efficiency with physical consistency, thereby paving the way for practical, real-time topology optimisation within digital-twin and generative-design environments.

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