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Full Length Research Paper

Development of a Rainfall-Runoff Model for the Calabar Metropolis Catchment Using the Differential Equation Approach

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Analytical approach involving differential equations was used to develop model that could predict the influence of some hydraulic and hydrologic parameters on the rainfall-runoff processes in Calabar Metropolis. The model generation relied on some other existing one-dimensional flow models.. The model was validated using the MAtlab v7 (R2008a) program. The RMSE approach was employed to compare the measured with the calculated discharges. Eight out of the ten locations gave strong results with the measured which ranged from 0.0370 to 0.625. There was a very strong correlation between the cross sectional area of drain and discharge (r = 0.93, p<0.01) as well as the basin area (r = 0.76, p<0.05). Incorrect sizing and spread of drains as well as the existing slopes employed in the generation of the drains' invert during construction have been seen as some of the key factors that foster flooding in the Metropolis. Misalignment of the drains with the existing outlet does not help the expected discharge of storm runoff to receiving bodies. Sequel to these, revisiting the Calabar Master Plan of 1972 with the original design for six drainage outlets is recommended. Designers and managers of the Calabar Metropolis catchment are strongly encouraged to make use of the developed model in the analysis of the rainfall-runoff processes within the Metropolis instead of the rule-of-thumb approach commonly employed.

Keywords: Rainfall-Runoff Model, Calabar Metropolis Catchment, Equation approach

INTRODUCTION

Hydrological model is a simplified representation of a natural system. It can be said that "a model" is a collection of symbols, which represents the system in a concise form that works as a representation of natural system or some aspects of it.

The rainfall- runoff model is one of the most frequently used events in hydrology. It determines the runoff signal which leaves the watershed from the rainfall signal received by the basin.

Rainfall- runoff modeling plays a pivotal supportive decision role in resolving practical water resource management and planning issues in any given watershed. In Calabar Metropolis for instance, after every storm event, some streets look clean -, while others look dirty. In both cases, there are problems. Unpredicted storms with its resultant runoff rushing over paved surfaces picks up wastes and pollutants from the clean surfaces to the dirty ones and then flows either directly or via storm systems, to the various water bodies in the Metropolis.

In Calabar Metropolis storm drains (and especially the main channels) may have been designed without the basic data, and may have relied on empirically – derived criteria as pointed out by Adeleye (1978). Yet studies of the channels as indicated by Effiong-Fuller (1998) and Ekeng (1998) have all shown that the channels only helped to alter the points of incidence of floods, while solving the problem only in a few areas. Rainfall- runoff

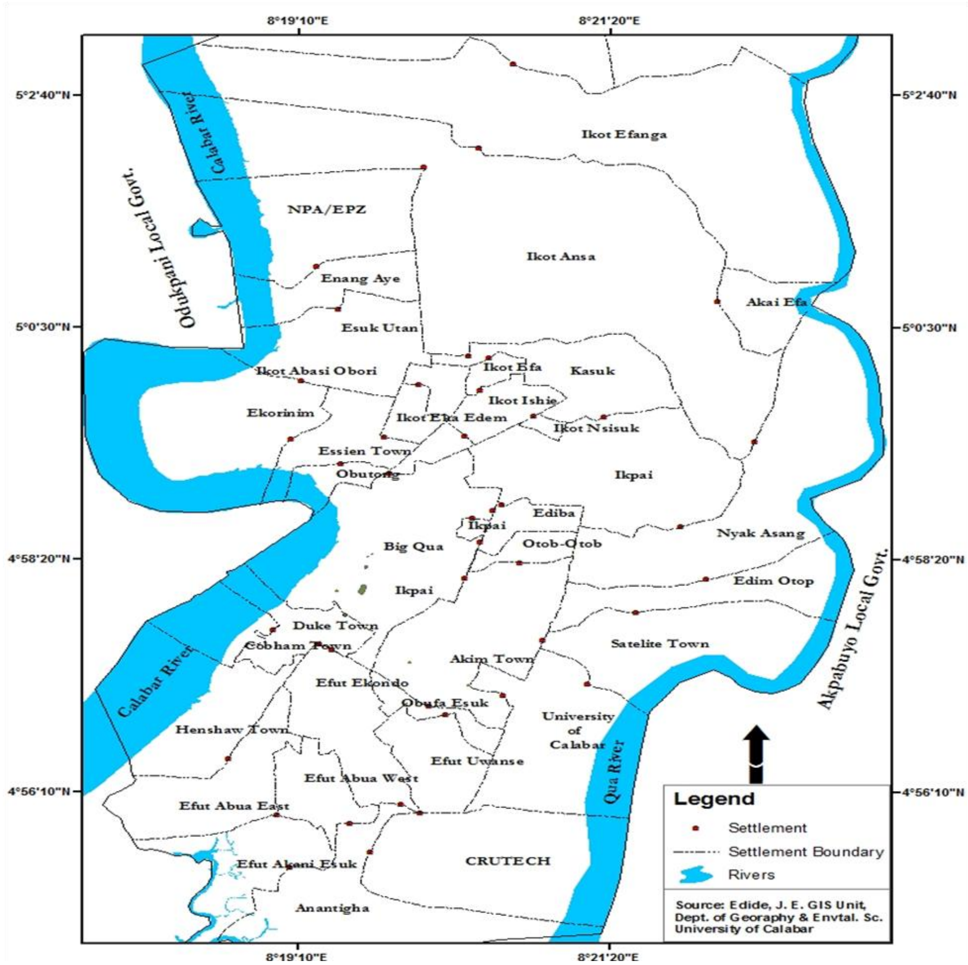

Figure 1: Layout of Calabar Metropolis

modeling plays a pivotal supportive decision role in resolving practical water resource management and planning issues in any given watershed. Calabar Metropolis has witnessed a very rapid urbanization over the same period. One of the many complex problems resulting from increased urbanization globally is related to management of storm water from developed areas. Proper water management has been a perennial problem in the Metropolis, as it is in many parts of the globe. This, of course has led to the incessant flooding of the streets and reduction of the quality of water in rivers and receiving water bodies. The overall objective of this research work is to develop a rainfall-runoff model for the prediction of all the components of rainfall-runoff processes in Calabar Metropolis drainage basin.

Specifically, the objectives of the research are to:

(i). Identify the pertinent factors or variables of the rainfall-runoff processes of the Calabar Metropolis catchment.

(ii). Develop a rainfall-runoff model based on these factors using differential equations.

(iii). Validate this model by comparing the computed results from the model with the measured data using the Matlab approach.

MATERIALS AND METHODS

i. Description of area of Study

Calabar Metropolis lies between latitudes 04° 45' 30" North and 05° 08'30" North of the Equator and longitudes 8° 11' 21" and 8°27'00" East of the Meridian. The town is flanked on its eastern and western borders by two large perennial streams viz: the Great Kwa River and the Calabar River respectively. These are aside from the numerous ephemeral channels which receive water after storm events to drain the area of study (Figure 1)

The Calabar River is about 7.58 metres deep at its two major bands (Tesko-Kutz, 1973). The city lies in a peninsular between the two rivers, 56km up the Calabar River away from the sea. Calabar has been described as an inter-fluvial settlement (Ugbong, 2000).

The present conditions as seen in terms of road network and settlements are as follows: The Calabar Road cum Murtala Muhammed Highway form the main artery of the city's roads network, running from north to south, linking all other major lines. Other major routes are the Ndidem Usang Iso Road, which runs parallel to the Highway, and MCC Road which runs perpendicular to both the Highway and Usang Iso Roads. Other streets spread like branches of a tree throughout the city.

The urban structure can best be explained in terms of the Hoytes (1939) as captured in Ugbong (2000) sectoral model. Population and settlements are concentrated in zones inhabited by the three ethnic groups-, the Efuts to the south, the Efiks to the west and the Quas to the east.

With a population of 202,585 in 1991, it now has a population of over 400,000, (C.R.S Ministry of Land and Housing, 2008).This shows a growth or an increase in population of 49.4% or an average annual population increase of 2.9%.

It occupies an area of about 223.325 sqkm with major clans being Efut Uwanse, Obufa – Esuk, Old Calabar, Mbukpa, Anantigha, Archibong Town, Cobham Town, Henshaw Town, Old Town, Essien Town, Ikot Ansa, Ikot Effanga, Ikot Omin, Ikot Nkebre, Akim Qua Town, Big Qua Town, Kasuk,Satellite Town, Nyakasang etc.

As a coastal town in Nigeria, Calabar metropolis has a high relative humidity, usually between 80% and 100%. Relative humidity drops with the rise in temperature to about 70% in the afternoon during the dry season. Vapour pressure in the air averages 29 millibars throughout the year (CRBDA Report, 1995).

All the year round, temperature rarely falls below 19^0C and average 27^0C. The average daily maximum is above 24^0C with a range of 6^0C, and a seasonal variation of the same amount, between the hottest month (March) and the coolest month (August). Expectedly therefore, evaporation will be high (Antigha, et al, 2014).

ii. Rainfall and Evapotranspiration Data

Rainfall measurements for the data were done at the Nigerian Meteorological Centre (NIMET) of the Margaret Ekpo International Airport and the Cross River University of Technology, Calabar, Cross River State.

Two sets of rainfall data were obtained for the study. The first was a twenty-eight (28) year daily/hourly rainfall data, while the second was a forty-three (43) year yearly/monthly rainfall data. The daily rainfall readings were obtained for twenty-three months from January, 2008 to November, 2009. These readings, out of the three hundred and thirty-three (333) months rain fell, were subjected to closer scrutiny. (This aligns with Wilson's (2006) 510 rainfall data from 17 different catchments in the U.K). This was because the runoff readings obtained from the area for the study were done during some of these storms' event. A total number of three hundred and forty six (346) storm events were recorded. Five thousand, five hundred and eighty seven millimeters (5587 mm) of rain was recorded for the period monitored. Total hours that rain fell were 1084.67hours. This gave an average rainfall of 242.9mm per month of rain, and an average intensity of 5.15mm per hour of rain. Both the cylindrical and self-recording rain guages were used for the rainfall readings.

Runoff Data

Flow measurements are critical to monitoring storm water best management practices (BMPs). Accurate flow measurements are necessary for accurate computing of samples used to characterize storm runoff and for the estimation of volumes. A total of ten (10) drainage outlets were selected as points for storm runoff readings. Twenty (20) storm events were monitored at each recording point and 80 (eighty) runoff readings were taken from each reading point with the propeller- type current meter model A.OTT, Kempton type F_4. The metre had a reading range of $n < 4.67$, v = 0.0560n + 0.040; $n \geq 4.67$, v = 0.0545n + 0.047 for propeller 1 and $n < 1.28$, v = 0.0905n + 0.040; $n \geq 1.2$, v = 0.1030n + 0.024 for propeller 2. The readings were taken during the months selected as the wettest part of the year (May to October) and for storms with duration of not less than 120 minutes (Darayatne, 2000).These gave a total of eight hundred (800) runoff readings. The readings were taken at the five minutes, ten minutes, and fifteen minutes up to the one hundred and twenty minutes rainfall intervals. These were recorded as $I_5 I_{10} I_{15}$ toI_{120} respectively. These pattern and intervals conform to the hydrologic standards, (State of New Jersey Urban Storm Drainage Design Manual, 2008; Storm Water Drainage Criteria Manual, 2004).

Model Development

To develop a model, it is important to define what purpose or purposes a model should have. Mulligan and Wainwright (2003) have identified three (3) purposes to which a general model is usually put. They included amongst others, an aid to research**,** a tool for simulation and prediction as well as a research product.

To aid the model development for the Calabar Metropolis catchment, the following assumptions were made.

- That, for a single storm event of average intensity, the length of the over land flow is, in a way proportional to the time of concentration which is taken as the time it will take for a storm runoff to travel from the inlet to the outlet of the sub basin or catchment.
- That,(as also opined by Dunne,1970 with modifications), half of the water that falls on the Calabar pluviometric surface as rain per given time, runs off as runoff after losses have been abstracted.

The basis of the model was a water balance process between:

- Input to the catchment as rainfall;
- Output from the catchment as runoff;
- Losses from the catchment as evapotranspiration.

The model development was done by the use of the St Venant Kinematic Wave Equation, the Manning's Equation, the Momentum Equation as well as the kostiakov equation. The water balance equation mentioned above may be summarized simply as

$P= E_t + Q + Q_{dp} + \Delta s$…………………………........... (1)

Where; P = rainfall., Et is evapotranspiration loss, Q is run-off, Qdp is deep percolation, Δs is change in storage.

However, the hydrological processes in a catchment of area (A) in m^2 are represented by the simple water balance theory given as

$\frac{ds}{dt} = p - e - q$ …………………………………………..(2)

Where; q = specific runoff = Q/A (mm/day), Q = runoff at the outlet in m^3/s, P = precipitation intensity in mm/day, E = rate of evaporation in mm/day, T = time in days, S = water storage in the area expressed as volume/catchment area in mm.

Over a long period, storage change is small and runoff can be estimated as p-e. The specific runoff is a function of the storage level and can be defined as

q= f(h)…………………………………………………(3)

This relationship changes with water level (h) changes and seasonal variations in catchment characteristics (Bengtsson, 1997). The water balance equation is modified by replacing S with 'h' to give;

$\frac{ds}{dt} = p - e - f(h)$……………………………………..(4)

This relationship represents a saturated catchment or catchment at field capacity with fast response and does not consider any of the physical variations that exist in reality. However, water storage and runoff changes with time and the relation are represented by

$q = f(h) = \frac{h}{T}$…………………………………………..…(5)

Here, T is the catchment time constant usually given as I/T implying that, the faster the catchment response to precipitation, the faster the runoff increases. The time constant depends on catchment size, topography and other catchment physical characteristics.

Overland Flow Approach

The overland flow is that portion of runoff that occurs as sheet over a land surface without becoming concentrated in well-defined channels, gullies and rills.

The model for overland flow based on the St. Venant Kinematic wave equation is given as

$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r_e$, ……………………………………………(6)

The kinematic wave theory relies on the continuity equation (i.e., conservation of mass) and a simplified form of the momentum equation to derive solutions for flow problems.

$q = \alpha h^{\beta}$ …………………………………………………..(7)

Where h is the depth of flow, r_e is the rainfall intensity, x is the variable representing space and t is the variable representing time space. Also, q is the flow per unit width, α , β are constants which can be obtained from the manning equation.

$Q = \frac{K_m}{n} AR^{2/3} S^{1/2}$ ………………………………………(8)

Where

K_m is a constant which is 1 in S.I unit, n is the roughness coefficient, A is the flow area R is the hydraulic radius which is the ratio of the flow area to wetted perimeter P, S is the surface slope.

Since $q = Vh$, then equation (6) becomes

$\frac{\partial h}{\partial t} + \frac{\partial Vh}{\partial x} = r_e$ ……………………………………….. (9)

$\frac{\partial h}{\partial t} + V\frac{\partial h}{\partial x} = r_e$ ………………………………………..(10)

For uniform flow, equation (7) which is the momentum equation is expressed as

$$q = \alpha h^{\beta}$$

Where C_k is the Kinematic wave celerity or speed for overland flow.

Then equation (6) becomes

$C_k \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = r_e$ …………………………………… (11)

To find out the analytic solution of the Kinematic wave equation in Equation (11) we use the method of characteristic of the first-order partial differential equation (PDES). The key idea in the method of characteristic is to change the coordinate system from (x,t) to a new one (x_0, s) in which the partial differential equation in (11) becomes an ordinary differential equation (ODE) along appropriate curves called the characteristic curve, in the $x-t$ plane. The new variable x_0 will be constant along the characteristics and will be point along the $t = 0$ axis in the $x-t$ plane. On the other hand, the new variable s will vary along the characteristic line. Using the form

$$h(x,t) = h[x(s),t(s)] \quad (12)$$

Where $[x(s),t(s)]$ is a characteristic line. Using chain rule, that

$$\frac{d}{ds}h[x(s),t(s)] = \frac{\partial h}{\partial x}\frac{dx}{ds} + \frac{\partial h}{\partial t}\frac{dt}{ds} \quad (13)$$

Compare this with Equation (11)

$$C_k \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = r_e \quad (14)$$

We can set, $\frac{dx}{ds} = C_k$ and $\frac{dt}{ds} = 1$ and $\frac{dh}{ds} = r_e$ ……….(15)

$$\frac{dx}{dt} = C_k = \alpha\beta h^{\beta-1} \quad (16)$$

$$dx = \alpha\beta h^{\beta-1} ds \quad (17)$$

Integrating both sides

$$\int dx = \int \alpha\beta h^{\beta-1} ds \quad (18)$$

$$x = \alpha\beta h^{\beta-1} S + C \quad (19)$$

With initial condition $t(0) = 0$, we have

$$0 = \alpha\beta h^{\beta-1}(0) + C \quad (20)$$

C = 0

Hence,

$$x = \alpha\beta h^{\beta-1} S \quad (21)$$

Since $\frac{dt}{ds} = 1$

$$dt = ds$$

$$\int dt = \int ds$$

$$t = S + C$$

But with the initial condition $t(0) = 0$

0 = 0 + c $\Rightarrow$ c = 0

Then $t = s + c = s + 0 = s$

t = s

Equation (21) then becomes

$$x = \alpha\beta h^{\beta-1} t. \quad (22)$$

If , $\frac{dh}{ds} = r_e$

$$dh = r_e ds \quad (23)$$

$$\int dh = r_e \int ds$$

$$h = r_e S + C \quad (24)$$

Applying the initial condition,

$$t(0) = 0$$

C = 0

$h = r_e S$, but t = s

$$h = r_e t \quad (25)$$

We can then write Equation (22) as

$$x = \alpha\beta (r_e t)^{\beta-1} t \quad (26)$$

$$x = \alpha\beta r_e^{\beta-1} t^{\beta} \quad (27)$$

Making t the subject

$$t = \left(\frac{x}{\alpha\beta r_e^{\beta-1}}\right)^{\frac{1}{\beta}} \quad (28)$$

But x = L is the distance, then

$$t = \left(\frac{L}{\alpha\beta r_e^{\beta-1}}\right)^{\frac{1}{\beta}} \quad (29)$$

$$t = \frac{L^{\frac{1}{\beta}}}{\left(\alpha_k r_e^{\beta-1}\right)^{\frac{1}{\beta}}} = \left(\frac{L}{\alpha_k r_e^{\beta-1}}\right)^{\frac{1}{\beta}} \quad (30)$$

Where t is the time required for a Kinematic wave to travel on overland flow of distance L or time of concentration.

Channel Flow Method

In the development of model for channel runoff, a new modified one-dimensional St. Venant Kinematic Wave equation was proposed. The equation is given as (Note that the detailed derivation of the model has been skipped).

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = f\,\frac{\phi}{2z_e}(r_e - T) \quad (31)$$

But $Q = V_k A$ and $f = az^b$ ………………………(32)

Where V_k is the celerity and A is the cross sectional area, then

$$A = \frac{Q}{V_k} \quad (33)$$

Substituting equation (34) into equation (32), gives the following,

$$\frac{\partial Q}{\partial x} + \frac{\partial}{\partial t}\left(\frac{Q}{V_k}\right) = \frac{a\phi}{2z_e}(r_e - T)z^b \quad (34)$$

$$\frac{\partial Q}{\partial x} + \frac{1}{V_k}\frac{\partial Q}{\partial t} = \frac{a\phi}{2z_e}(r_e - T)z^b \quad (35)$$

Using the method of characteristics to solve for V_k

$$Q(x,t) = Q[x(s),t(s)]$$

$$\frac{d}{ds}Q[x(s),t(s)] = \frac{\partial Q}{\partial x}\frac{dx}{ds} + \frac{\partial Q}{\partial t}\frac{dt}{ds} \quad \text{.....................(36)}$$

Comparing equation (37) with (36), the following relationship is obtained

$$\frac{dx}{ds} = 1 \text{ and } \frac{dt}{ds} = \frac{1}{V_k} \quad \text{....................................(37)}$$

$$\frac{dt}{ds} = \frac{1}{V_k}, \Rightarrow \frac{dQ}{ds} = \frac{a\phi}{2z_e}(r_e - T)z^b \quad \text{......................(38)}$$

$$ds = V_k dt \quad \text{...(39)}$$

Integrating both sides

$$S = V_k t + c \quad \text{..(40)}$$

Using the initial condition t (0) = 0, c = 0

Then

$$S = V_k t \quad \text{..(41)}$$

$$V_k = {}^{s}\!/_{t} \quad \text{..(42)}$$

But $\frac{dx}{ds} = 1$ ....43)

$$dx = ds$$

$$\int dx = \int ds$$

$$x = S + C$$

Where C is the integral constant

Also, using initial condition

$$x(0) = 0, C = 0$$

$x = 0$ and $x = L \Rightarrow S = L$

Then $V_k = {}^{s}\!/_{t} = \frac{L}{t}$

Therefore, the celerity V_k can be obtained the formula

$$V_k = \frac{L}{t} \quad \text{... (44)}$$

Where L is the channel length and t is travel time to the outlet. An expression for t has been derived from overland flow model as

$$t = \left(\frac{L}{\alpha r_e^{\beta-1}}\right)^{\frac{1}{\beta}} \quad \text{...(45)}$$

Where α can be obtained from equations depending on the shape of the channel.

$\beta = \frac{5}{3}$ and r_e is the rainfall intensity

From the foregoing, the model derived therefore is given as,

$$\frac{\partial Q}{\partial x} + \frac{1}{V_k}\frac{\partial Q}{\partial t} = \frac{a\phi}{2z_e}(r_e - T)z^b \quad \text{...........................(46)}$$

Where

Δt is change in time,

Δx is change in length along the path of flow(from inlet to outlet),

ϕ is a catchment constant defining the channel finish.

V_k is the celerity,

Q is the discharge,

r_e is the rainfall intensity

T is the catchment losses (evapotranspiration).

z is the time from the onset of infiltration

α and b are infiltration constants.

z_e is a time constant assumed as unity.

The solution to this model was done using the numerical approach of solving partial differential equation. The finite difference method was used. Finite difference formulation for runoff through the channel is given as

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{1}{V_k}\frac{Q_{i+1}^{j+1} - Q_{i+1}^{j}}{\Delta t} = \frac{a\phi}{2ze}(r_e - T)z^b \quad \text{...........,...(47)}$$

$$\frac{v_k \Delta t\left(Q_{i+1}^{j+1} - Q_i^{j+1}\right) + \Delta x\left(Q_{i+1}^{j} - Q_i^{j+1}\right)}{\Delta t \Delta x v_k} = \frac{a\phi}{2z_e}(r_e - T)z^b \quad \text{..........(48)}$$

$$v_k \Delta t\left(Q_{i+1}^{j+1} - Q_i^{j+1}\right) + \Delta x\left(Q_{i+1}^{j} - Q_i^{j+1}\right) = \frac{a\phi}{2z_e}(r_e - T)z^b v_k \Delta x \Delta t \quad \text{......}$$

....(49)

$$v_k \Delta t Q_{i+1}^{j+1} - v_k \Delta t Q_i^{j+1} + \Delta x Q_{i+1}^{j+1} - \Delta x Q_i^{j+1} = \frac{a\phi}{2z_e}(r_e - T)z^b v_k \Delta x \Delta t$$

....(50)

$$v_k \Delta t Q_{i+1}^{j+1} + \Delta x Q_{i+1}^{j+1} = v_k \Delta t Q_i^{j+1} + \Delta x Q_i^{j+1} + \frac{a\phi}{2z_e}(r_e - T)z^b v_k \Delta x \Delta t$$

....(51)

$$Q_{i+1}^{j+1}(v_k \Delta t + \Delta x) = Q_i^{j+1}(v_k \Delta t + \Delta x) + \frac{a\phi}{2z_e}(r_e - T)z^b v_k \Delta x \Delta t$$

....(52)

Dividing both sides by $(v_k \Delta t + \Delta x)$ to make Q_{i+1}^{j+1} the subject, yields the following,

$$Q_{i+1}^{j+1} = Q_i^{j+1} + \frac{\frac{a\phi}{2z_e}(r_e - T)z^b v_k \Delta x \Delta t}{v_k \Delta t + \Delta x}$$

....(53)

Where Δt the change in time space (time step), Δx is the change in distance, $\phi = 0.17$, i and j denote increment time and space level respectively.
To be able to solve this equation, we use the initial condition t = 0, Q = 0 , A = 0 and the boundary condition is zero inflow which means that Q = 0 and A = 0.
$\Delta t =$ 5 minutes = 300 sec, t = 0,5, 10, ….,120
The next step is to use the MATLAB approach in generating the code which is to be applied to equation (51) for the final models for each station and channel shape.

Table 1: Various locations and the Measured Variables

S/N	Area Name	Basin Area (ha)	Sum of Channel Length (m)	Artificial Drainage Density (Dd)	Cross Sectional Area of drain(m^2)	Measured Discharge ($\frac{m^3}{s}$)	Degree of impervious Area (%)	Gradient (m/m)
1	Ediba One Area	179.4	2978.0	16.6	0.88	4.85	67.01	0.010
2	Ediba two Area	274.8	3305.24	12.03	2.125	13.0	65.59	0.010
3	Ibom Layout Area	189.3	2478.92	13.1	0.63	1.80	69.0	0.017
4	Mayne Avenue Area	280.2	1970.21	7.03	0.63	1.94	65.4	0.006
5	Big Qua Area	193.6	1579.0	8.16	0.33	0.69	66.24	0.010
6	M.C.C. Highway Area	559.8	4543.02	8.12	6.075	32.2	63.0	0.011
7	Yellow-Duke/Inyang Area	221.0	3611.15	16.34	0.556	1.94	64.9	0.005
8	Marina Road Area	213.5	2195.36	10.28	3.08	9.7	63.95	0.0114
9	Marian Road Area	301.4	2780.01	9.22	0.22	0.66	72.3	0.023
10	Mary Slessor	406.8	3478.07	8.55	1.4	4.40	66.2	0.016

RESULTS AND DISCUSSIONS

Tables 1 and 2, as well as figure 2 show the measured and calculated results as obtained from the catchment.

Table 2: Measured Discharge, Velocity, Flow Depth per time of Measurement for Locations 1-10

Locations	Δt- time interval (Mins)	ΔV- velocity (m/s)	Δh-depth (m)	Cross Sectional Area (A) (m^2)	ΔQ (discharge) measured m^3/s
Location I	5 min	1.52	0.26	0.88	1.34
	10 min	3.10	0.33	0.88	2.73
	15 min	4.82	0.38	0.88	4.24
	20 min	5.65	0.42	0.88	4.97
	30 min	9.79	0.58	0.88	8.62
	40 min	10.69	0.61	0.88	9.41
	50 min	7.63	0.45	0.88	6.71
	60 min	5.32	0.40	0.88	4.68
	90 min	4.15	0.36	0.88	3.65
	120min	2.42	0.37	0.88	2.13
Location 2	5 min	1.77	0.44	2.125	3.76
	10 min	3.62	0.48	2.125	7.69
	15 min	5.01	0.56	2.125	10.65
	20 min	7.04	0.61	2.125	14.96
	30 min	10.32	0.71	2.125	21.93
	40 min	11.57	0.85	2.125	24.59
	50 min	8.60	0.65	2.125	18.28
	60 min	6.23	0.59	2.125	13.24
	90 min	4.25	0.52	2.125	9.03
	120 min	2.7	0.46	2.125	5.74
Location 3	5 min	0.78	0.12	0.63	0.49
	10 min	1.64	0.14	0.63	1.03
	15 min	2.28	0.17	0.63	1.44
	20 min	3.28	0.23	0.63	2.07
	30 min	4.80	0.49	0.63	3.02
	40 min	5.40	0.61	0.63	3.40
	50 min	3.90	0.54	0.63	2.46
	60 min	2.66	0.19	0.63	1.68
	90 min	2.03	0.15	0.63	1.28
	120 min	1.16	0.13	0.63	0.73
Location 4	5 min	0.82	0.36	0.63	0.52
	10 min	1.74	0.45	0.63	1.10
	15 min	2.49	0.50	0.63	1.57
	20 min	3.52	0.58	0.63	2.22
	30 min	5.40	0.62	0.63	3.40
	40 min	5.72	0.65	0.63	3.60
	50 min	3.0	0.55	0.63	2.84
	60 min	2.20	0.53	0.63	1.89
	90 min	1.03	0.47	0.63	1.39
	120 min	1.16	0.42	0.63	0.82
Location 5	5 min	0.59	0.15	0.33	0.19
	10 min	1.21	0.22	0.33	0.40
	15 min	1.84	0.27	0.33	0.61
	20 min	2.68	0.41	0.33	0.88
	30 min	3.77	0.62	0.33	1.24
	40 min	3.67	0.50	0.33	1.21
	50 min	2.68	0.43	0.33	0.88
	60 min	2.16	0.32	0.33	0.71
	90 min	1.49	0.25	0.33	0.49
	120 min	1.0	0.19	0.33	0.33

Table 2: Measured Discharge, Velocity, Flow Depth per time of Measurement for Locations 1-10 (**Continue**)

Locations	**Δt- time interval (Mins)**	**ΔV- velocity (m/s)**	**Δh-depth (m)**	**Cross Sectional Area (A) (m^2)**	**ΔQ (discharge) measured m^3/s**
Location 6	5 min	1.77	0.21	6.075	10.75
	10 min	3.10	0.25	6.075	18.83
	15 min	4.34	0.32	6.075	27.58
	20 min	6.23	0.83	6.075	37.84
	30 min	9.88	1.14	6.075	60.02
	40 min	9.27	1.08	6.075	56.32
	50 min	7.25	0.97	6.075	44.04
	60 min	5.22	0.52	6.075	31.71
	90 min	3.64	0.29	6.075	22.11
	120 min	2.10	0.23	6.075	12.76
Location 7	5 min	0.95	0.31	0.556	0.53
	10 min	1.92	0.37	0.556	1.08
	15 min	3.09	0.46	0.556	1.72
	20 min	4.62	0.60	0.556	2.57
	30 min	6.41	0.68	0.556	3.56
	40 min	5.93	0.65	0.556	3.30
	50 min	4.80	0.62	0.556	2.67
	60 min	3.20	0.51	0.556	1.78
	90 min	2.36	0.42	0.556	1.31
	120 min	1.59	0.34	0.556	0.87
Location 8	5 min	0.93	0.11	3.08	2.86
	10 min	1.74	0.15	3.08	5.36
	15 min	2.62	0.19	3.08	8.07
	20 min	3.63	0.46	3.08	11.18
	30 min	5.48	0.82	3.08	16.88
	40 min	5.98	0.94	3.08	18.42
	50 min	4.61	0.79	3.08	14.20
	60 min	2.99	0.32	3.08	9.21
	90 min	2.24	0.16	3.08	6.90
	120 min	1.27	0.14	3.08	3.91
Location 9	5 min	0.79	0.21	0.22	0.17
	10 min	1.70	0.29	0.22	0.37
	15 min	2.45	0.38	0.22	0.54
	20 min	3.32	0.46	0.22	0.73
	30 min	4.90	0.58	0.22	1.08
	40 min	5.54	0.74	0.22	1.22
	50 min	4.36	0.52	0.22	0.96
	60 min	3.12	0.44	0.22	0.69
	90 min	2.29	0.32	0.22	0.50
	120 min	1.43	0.26	0.22	0.31
Location10	5 min	0.92	0.23	1.4	1.29
	10 min	1.94	0.26	1.4	2.72
	15 min	2.70	0.31	1.4	3.78
	20 min	3.80	0.36	1.4	5.32
	30 min	5.22	0.54	1.4	7.31
	40 min	5.79	0.85	1.4	8.11
	50 min	4.40	0.42	1.4	6.16
	60 min	3.05	0.33	1.4	4.27
	90 min	2.28	0.27	1.4	3.19
	120 min	1.35	0.24	1.4	1.89

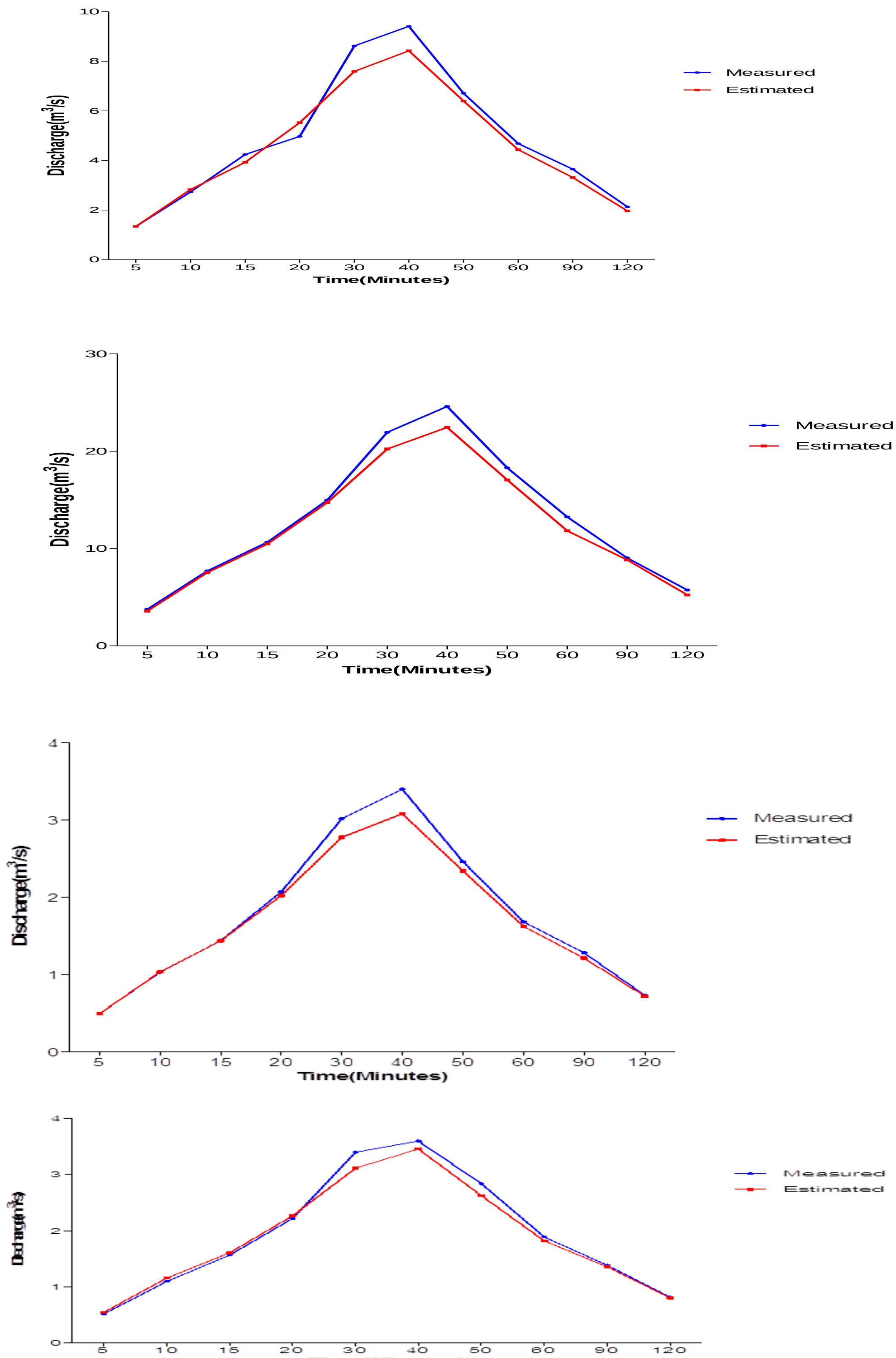

Figure 2: Plots of measured and calculated discharge for some locations in the study area.

From the validation using the plotted discharge against time (fig.2), there was a marked closeness between the measured and the calculated discharge in most locations. For example, the plotted difference between the measured and the calculated for locations 4,5 and 7 respectively were 0.56, 0.51 and 1.33 respectively. RMSE for locations 3,4,5,9 and 10 are 0.1365, 0.1267, 0.0703, 0.0935 and 0.1902 respectively. Locations 1, 2,7 and 8 gave values that were, though not high, but were considered not to be as good as the previous five locations mentioned. They had the following values respectively, 0.3258, 0.1259 and 0.1844. The RMSE for location 6 was high, above 2.6. This observed calculated error may tangentially be connected with the size of the drain and the flow it conveys. The programme may not have been able to properly or correctly simulate the channel's flow. It should be noted here that location 6 is the inlet from where channel 1, the No. 1 discharge route in the metropolis takes its flow. This however is aside from a possible input error during the running of the program

The difference between the measured and the calculated from location 1 to 10 ranged from -0.74 to 8.03. From the results, it was observed that the measured values were just slightly higher than the calculated values for only locations 6 and 2. This, as said earlier, could be expected as there is every tendency of slight input errors during program running. In general, there was an agreement of the measured with the calculated for most locations.

The results of the bivariate correlation analysis to determine which of these variables were significantly associated with the discharge showed a significant positive relationship between discharge and basin area ($r = 0.76$, $p<0.05$), sum of channel length and discharge ($r = 0.67$, $p<0.05$), cross sectional area and discharge ($r = 0.93$, $p<0.05$) as well as length of overland flow ($r= 0.68$, $p<0.05$). Also, based on the results of the factor analysis, basin area, sum of channel length, cross sectional area and length of overland flow were shown as the major variables that affect discharge in the study area.

The developed model was given as

$$\frac{\partial Q}{\partial x}+\frac{1}{V_k}\frac{\partial Q}{\partial t}=\frac{a\phi}{2z_e}\left(r_e-T\right)z^b$$

Where

Δt is change in time, Δx is change in length along the path of flow(from inlet to outlet),

ϕ is a catchment constant defining the topography, geology, channel finish etc, *Vk* is the celerity, *Q* is the discharge, *re* is the rainfall depth and *T* is the catchment losses (evapotranspiration). The left hand side of the model simulates the non-uniform and unsteady flow aspects of the catchment flow (i.e., spatial and temporal variation of flow) respectively, while the right hand side addresses the catchment characteristics as well as the climatic components of the catchment. However, according to Aron (1973), with respect to momentum, flow is assumed to be steady and uniform from one time increment to the next.

CONCLUSION AND RECOMMENDATIONS

In the present water and environmental developmental requirements, especially in the developing countries like Nigeria, where cases of either poorly gauged or completely ungauged basins proliferate, there is a discernible demand for the application of a synthetic but verifiable hydrologic procedure. The differential equation approach was used to develop a conceptual model that incorporates a water balance concept to investigate the rainfall-runoff relationship and the model adequacy and application within Calabar catchment. The validation of the measured with the calculated using the MatLab approach gave a good correlation, showing that the developed model is applicable in the catchment.

In urban storm drainage systems studies, rainfall-runoff processes are normally analysed by the application of mathematical models sometimes in combination with other various water quantity and quality sampling techniques. Urbanization has been shown to increase surface runoff, by creating more impervious surface such as pavement and structures that impede percolation. When this happens, the water instead is forced to flow directly into streams or storm water runoff drains, where erosion and siltation can be major problems, even when flooding is not.

It is important to note that the amount of storm water runoff present at any given point in time in an urban watershed cannot be compressed or diminished. Open and enclosed storm systems serve both conveyance and storage functions. If adequate provision is not made for drainage space demands, storm water runoff will normally conflict with other land uses, thereby resulting in damage to public and private property as well as impairment or disruption of other urban systems.

Studies have shown that in urban watersheds that have been developed without adequate storm water planning, there is generally inadequate space available to construct detention storage facilities to reduce peak flows significantly along major waterways. Attempts to create adequate space to construct such storage facilities will generally require the removal of valuable existing facilities and this, of course is often not economically or socially feasible.

It is recommended that the design of the storm water drainage system should carefully consider the need for accessibility and maintenance to sustain adequate function. Failure to provide proper maintenance reduces both the hydraulic capacity and the pollutant removal efficiency of the system.

Additionally, for the Calabar Metropolis urban storm water drainage system, further planning and design of drainage facilities should include consideration of scheduling of work crews and funding necessary to provide proper maintenance. This will in no small measure ensure proper workability of the drainage systems in the metropolis.

The model so developed is for the study and design of rainfall-runoff processes is the metropolis. Designers are encouraged to apply the model in their subsequent drainage network design jobs in the metropolis. This will in no small measure check the incidences of under-designing of drains and improper alignment.

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